Math Virtual Learning

Calculus AB

Integrate natural log, exponential, and inverse trig functions

April 10, 2020

Calculus AB
Lesson: April 10, 2020

Objective/Learning Target:

Students will integrate natural log, exponential, and inverse trig functions

Warm-Up:

Today is a review of everything we have covered this week. So to start go back and review the lessons from April 6-9.

Videos: Natural Logs
Exponentials with base e
Exponentials with integer base
Inverse Trig

Practice:

We will look at one of each type of problem. Remember to go back to this weeks other lessons for more worked out examples.

Ex 1:

Calculate the integral

$$\int \frac{x}{x^2+4}dx.$$

Solution

Using u-substitution, let $u = x^2 + 4$. Then $du = 2xdx$ and we have

$$\int \frac{x}{x^2+4}dx = \frac{1}{2}\int \frac{1}{u}du\frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|\,x^2+4\,| + C = \frac{1}{2}\ln(x^2+4) + C.$$

Ex 2:

$$\int \frac{2}{e^{3x}}dx$$

$$= \int 2e^{-3x}dx$$

$$= \frac{2e^{-3x}}{-3} + C$$

$$= \frac{-2e^{-3x}}{3} + C$$

Practice (continued):

3) Integrate $\int 7^{2x+3}\,dx$. Use u-substitution. Let

$$u = 2x+3$$

so that

$$du = 2\,dx\,,$$

or

$$(1/2)\,du = dx\,.$$

Substitute into the original problem, replacing all forms of x, getting

$$\int 7^{2x+3}\,dx = \int 7^{u}\,(1/2)du$$

$$= (1/2)\int 7^{u}\,du$$

(Now use formula 2 from the introduction to this section on integrating exponential functions.)

$$= (1/2)\frac{7^{u}}{\ln 7} + C$$

(Recall that $A\ln B = \ln B^{A}$.)

$$= \frac{7^{2x+3}}{\ln 7^{2}} + C$$

$$= \frac{7^{2x+3}}{\ln 49} + C\,.$$

Practice (continued):

4) Evaluate the integral $\int \frac{dx}{\sqrt{4-9x^2}}$.

Substitute $u = 3x$.

Then $du = 3dx$

and we have

$$\int \frac{dx}{\sqrt{4-9x^2}} = \frac{1}{3}\int \frac{du}{\sqrt{4-u^2}}.$$

Applying the formula with $a = 2$,

we obtain

$$\int \frac{dx}{\sqrt{4-9x^2}} = \frac{1}{3}\int \frac{du}{\sqrt{4-u^2}}$$

$$= \frac{1}{3}\sin^{-1}\left(\frac{u}{2}\right) + C$$
$$= \frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right) + C.$$

Practice: Evaluate the following.

1) $\displaystyle\int \frac{x^2}{x^3+6}\,dx.$

2) $\displaystyle\int e^{7x-2}\,dx$

3) Suppose the rate of growth of bacteria in a Petri dish is given by $q(t) = 3^t$, where t is given in hours and $q(t)$ is given in thousands of bacteria per hour. If a culture starts with 10,000 bacteria, find a function $Q(t)$ that gives the number of bacteria in the Petri dish at any time t. How many bacteria are in the dish after 2 hours?

4) Find the antiderivative of $\displaystyle\int \frac{dx}{\sqrt{1-16x^2}}.$

Answer Key:

Once you have completed the problems, check your answers here.

1) $$\int \frac{x^2}{x^3+6}dx = \frac{1}{3}\ln|x^3+6| + C$$

2) $$\int e^{7x-2}dx = \frac{e^{7x-2}}{7} + C$$

3) We have

$Q(t) = \int 3^t dt = \frac{3^t}{\ln 3} + C.$

Then, at $t = 0$ we have $Q(0) = 10 = \frac{1}{\ln 3} + C$, so $C \approx 9.090$ and we get

$Q(t) = \frac{3^t}{\ln 3} + 9.090.$

At time $t = 2$, we have

$Q(2) = \frac{3^2}{\ln 3} + 9.090$

$= 17.282.$

After 2 hours, there are 17,282 bacteria in the dish.

4) f $$\int \frac{dx}{\sqrt{1-16x^2}}$$

$$\frac{1}{4}\sin^{-1}(4x) + C$$

Additional Practice:

Review Worksheet with Answers #1

Review Worksheet with Answers #2